Dynamic Coordination With Network Externalities: Procrastination Can Be Efficient

Deivis Angeli *

São Paulo School of Economics - FGV, University of British Columbia

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How does present bias affect welfare when agents want to coordinate over time? To answer that, I analyze a dynamic coordination model under quasi-hyperbolic discounting. I document a novel mechanism through which present bias can be adaptive, i.e., it can internalize the social cost of coordinating on a new action, say going from coordinating on using Twitter to using Threads. Agents migrating from Twitter to Threads ignore that their choice imposes negative externalities on those still using Twitter. So, to achieve efficiency, regular exponential discounters should ask for a higher relative quality of Threads before adopting it. In turn, present biased agents overvalue the externalities they currently receive from Twitter since externalities from Threads can only come in the future, after others adopt it. Hence, present bias leads agents to ask for more quality before migrating to Threads, preventing paths of inefficient coordination. Furthermore, small amounts of present bias always prevent society from taking inefficient paths.

Keywords: dynamic coordination, network effects, technology adoption, present bias, quasi-hyperbolic discounting, welfare

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*deivisangeli@gmail.com – 6000 Iona Dr, Vancouver, BC V6T 1L4, Canada

Introduction

Problems of dynamic coordination are common – in many situations, an action becomes more attractive as it becomes popular. For instance, take social networks. If most of your friends were on Facebook rather than in the late Google +, then even if Google + had better functionalities, Facebook could be preferred if you do not expect your friends to migrate any time soon. Another famous example was the adoption of QWERTY keyboards instead of possibly more efficient standards (David, 1985).

Since externalities are part of these problems by definition, the resulting equilibrium can be inefficient. We might expect coordination on a lower-intrinsic-quality technology like QWERTY to be inefficient. However, Guimaraes and Pereira (2016) (henceforth GP) shows that when agents are regular exponential discounters, a central planner would suggest agents demand *more* intrinsic quality from the alternative keyboard before adopting it – implying that if we observe coordination on QWERTY, it is efficient. In general, an equilibrium can be inefficient because when individuals consider changing keyboards, they do not account for the decrease in the externality benefiting those have not changed keyboards yet.[1]

At the same time, an extensive literature has studied and documented the regularity with which economic agents are affected by present bias. Present-biased agents over-value their present happiness when making temporal trade-offs. For instance, they procrastinate on taking a costly action that yields a future benefit, even if ex-ante they would prefer immediate action. This kind of behavior has been studied in multiple contexts, such as distributing effort over time, consumption, and exercising (Ericson and Laibson, 2019). So, since agents in dynamic coordination games mainly make temporal trade-offs, and efficient behavior can be counter-intuitive (as argued above), it is important to know the implications of present bias for welfare. To make it concrete, consider the trade-off faced by a potential early adopter of an unpopular action: that person has to weigh the extra happiness coming from more intrinsic quality (from now on) against a (temporary) decrease in the received externalities. If the decrease in externalities is indeed temporary, present biased agents should ask for more intrinsic quality before becoming early adopters. And this suggests that a present biased society behaves more like a central planner would suggest.

This paper studies the welfare implications of introducing present bias in dynamic coordination problems by modifying the GP framework to allow for quasi-hyperbolic preferences. I focus on an overlapping generations version of the model in which individuals (newborns) choose their actions when they enter the

[1] In this paper's framework, an equlibrium is inefficient if individuals take actions leading to an adoption path different from what the planner would prescribe.

economy.[2] The main result confirms the laid out intuitions: present bias nudges individuals towards behavior the central planner would choose, so, in a present biased society, the set of inefficient equilibria is restricted. Moreover, as behavior changes continuously with the introduction of present bias, I find that a small amount of present bias is helpful without downsides.

If present bias is too pronounced, it may overshoot the mark, making a society coordinate in an inferior action when it should not. Hence, I also study under which conditions present bias leads to preventing inefficient choices and when it does not. Put another way, when does present bias flip the central planner's policy recommendation from *wait for more quality* to *adopt it now*? Intuitively, I find that present bias is more likely to be helpful when the coordination issue is more pronounced, i.e., when the long-term discount rate is high and when the population can not change actions quickly.

An important caveat is that while my model allows for variation of the relative intrinsic quality parameter over time, the main results are for the case where its variation is small. It effectively focuses on the case where the externalities are most important, but it ignores, for instance, cases where agents expect the competing network to get significantly intrinsically better over time. In this latter case, present bias should not be as helpful since agents would overly discount future utility coming from increases in quality, which is unrelated to the externalities.

By exploring how present bias affects dynamic coordination and welfare in such situations, this paper contributes to the study of behavioral biases interacted with markets and strategic play (e.g., Akerlof (1991); Laibson (1997); Kleinberg et al. (2017); Heidhues and Kőszegi (2010, 2017); Gans and Landry (2019); Bohren and Hauser (2021)). It also speaks to the question of how present bias can be rationalized, and instead of individual-based explanations (e.g., Chakraborty (2021); Dasgupta and Maskin (2005)), this paper finds that decision-making in groups can also generate situations where present bias is a beneficial trait. Finally, it also speaks to a broader literature on dynamic coordination problems which includes applications to business cycles (Frankel and Burdzy, 2005; Guimaraes and Machado, 2018), technology adoption (Crouzet et al., 2019), currency trading (Plantin and Shin, 2018), and neighborhood choice (Frankel and Pauzner, 2002).

I organize the rest of the paper as follows. In Section 5, I present the model and derive its equilibria with a generic utility function under quasi-hyperbolic discounting. Section 2 describes the results for a linear utility model, contextualizing it in the coordination problem of choosing QWERTY vs. the Dvorak Simplified Keyboard (DSK) as the keyboard standard (as in GP). Using the results from section 2, I tackle

[2] The equilibria from the overlapping generations model are also equilibria for a formulation with infinitely-lived agents, as remarked in Section 3. In that case, instead of being substituted at a certain rate, agents get chances to rethink their choices at the same rate.

the planner problem in Section 3, where I also show that present bias continuously moves the equilibrium in the same direction as the planner would prescribe. Section 4 explores when present bias can be helpful and finds that present bias stands a better chance of preventing inefficient outcomes when the long-term discount rate is high and the substitution rate is low. Section 5 concludes.

1 Set Up and Equilibrium Existence

This section introduces the model and describes the possible equilibria with a generic utility function. See the Appendix for a more through exposition and proofs. For a more comprehensive introduction to the dynamic coordination framework with a similar notation as in this paper, see Guimaraes et al. (2020).

I consider an overlapping generations formulation. Agents choose their actions when they enter the game. Lifespans have exponential distribution with average δ^{-1}, so δ is the rate at which agents are substituted (henceforth substitution rate). The results will also hold for the case with a fixed measure of (sophisticated or naive) infinitely lived agents who receive chances to rethink their choices at the same rate δ, as argued in Section 3. Most results follow almost immediately from Frankel and Pauzner (2000) (henceforth FP) and Burdzy et al. (1998).

1.1 The Discount Function

I model present bias using a quasi-hyperbolic discount function. From the point of view of an agent at time τ, instantaneous utility at time t is $D(t,\tau)u_t$, where D is the discount function and u_t is the flow utility. In discrete time models, it is common to use discount functions such as $\beta^{\mathbb{1}_{t>\tau}}\Delta^{t-\tau}$, with $(\beta,\Delta)\in[0,1]^2$. When $\beta\in[0,1)$, we have present bias.[3] While there are multiple ways of extending this to continuous time, for simplicity, I use

$$D(t,\tau)=\beta^{\mathbb{1}_{t-\tau\geq 1}}e^{-\rho(t-\tau)}$$

, where $\mathbb{1}$ is the indicator function. In the discount function D, I normalize the length of the present to 1. From the point of view of an agent at time τ, all utility incurred after one unit of time is discounted by β. The length of the present should be thought of as very short (see for instance Ericson and Laibson (2019)

[3] This discount function was introduced by Phelps and Pollak (1968) and widely used later on. See Cohen et al. (2020) for a review of other models of present bias.

and Frederick et al. (2002)). When $\beta = 1$, we are left with regular exponential discount at rate ρ.[4]

1.2 The Dynamic Coordination Problem

Time is continuous and homogeneous agents choose between two actions. There is a continuum of individuals $i \in [0,1]$, with lifespans with cumulative density $F(s) = 1 - e^{-\delta s}$. Newborns choose $a_{it} \in \{0,1\}$ once and are substituted when they die. The flow utility of changing to action 1 is $\Delta u(\theta_t, n_t)$, which is continuously differentiable and increasing in both arguments. That is, the benefit of choosing action 1 over 0 increases with the intrinsic relative quality of action 1, θ_t, and with the number of individuals taking action 1, n_t. A newborn at time at time τ with discount function D is uncertain about the path of (θ_t, n_t). So she (the agent) chooses 1 if

$$\mathbb{E}\left[\int_{\tau}^{+\infty} e^{-\delta(t-\tau)} D(t,\tau) \Delta u(\theta_t, n_t)\, dt\right] > 0. \tag{1}$$

1.2.1 Model With No Shocks

Without shocks, we can find two threshold functions:

These functions solve:

$$\int_{\tau}^{+\infty} e^{-\delta(t-\tau)} D(t,\tau) \Delta u(\theta^{opt}(n_\tau), n_\tau^{\uparrow}(t))\, dt = 0 \tag{2a}$$

$$\int_{\tau}^{+\infty} e^{-\delta(t-\tau)} D(t,\tau) \Delta u(\theta^{pes}(n_\tau), n_\tau^{\downarrow}(t))\, dt = 0 \tag{2b}$$

where $n_\tau^{\uparrow}(t) = 1 - (1 - n_\tau)e^{-\delta(t-\tau)}$ is the most optimistic path for action 1 and $n_\tau^{\downarrow}(t) = n_\tau e^{-\delta(t-\tau)}$ is the most pessimistic. The dashed lines in Figure 1 are linear representations of these thresholds.

1.2.2 Model With Shocks

Now let shocks follow a random walk: $d\theta_t = \mu\, dt + \sigma dZ_t$, where Z_t follows a standard Brownian motion. Then we have:

Proposition 1 *Let the trend from the Brownian motion, μ, be a constant. Assume again that dominance regions exist. There is a unique decreasing function $\theta^*(n_\tau)$ such that if $\theta_\tau < \theta^*(n_\tau)$, in equilibrium everyone chooses 0. Conversely, if $\theta_\tau > \theta^*(n_\tau)$, everyone chooses 1.*

[4]See Webb (2016) for more on modelling present bias in continuous time. The results presented here are similar if we use $D(t,\tau,\lambda) = \begin{cases} e^{-(\hat{\beta}/\lambda+\rho)(t-\tau)} & \text{if } t-\tau \leq \lambda \\ e^{-\hat{\beta}-\rho(t-\tau)} & \text{if } t-\tau > \lambda \end{cases}$, with λ as the length of the present.

This result stems from a contagion argument triggered by the randomness in the path of the relative quality parameter. Complete proofs are found in FP and Guimaraes et al. (2020).

1.2.3 Small Shocks

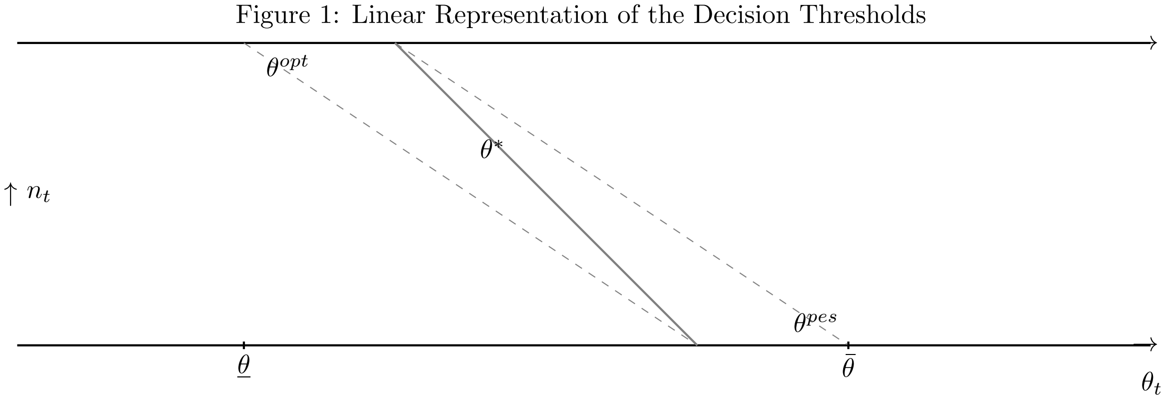
Proposition 8 shows that a unique threshold exists under frequent shocks, and now I focus on the case where one can directly calculate that threshold with small shocks.

Proposition 2 *Let* $d\theta_t = \alpha\psi(t,\theta_t)\,dt + \sigma dZ_t$. *If* $(\alpha,\sigma^2) \to \mathbf{0}$, *then the threshold* θ^* *solves:*

$$(1-n_\tau)\int_\tau^{+\infty} e^{-\delta(t-\tau)} D(t,\tau)\Delta u(\theta^*(n_\tau), n_\tau^{\uparrow}(t))\,dt + n_\tau \int_\tau^{+\infty} e^{-\delta(t-\tau)} D(t,\tau)\Delta u(\theta^*(n_\tau), n_\tau^{\downarrow}(t))\,dt = 0 \qquad (3)$$

While the proof follows the same steps as in FP and relies on mathematical results from Burdzy et al. (1998), it is worth understanding its intuition. First, notice that if you start on a point on the unique threshold, $(\theta^*(n_\tau), n_\tau)$, while θ_t moves, it does so slowly when $(\alpha,\sigma^2) \to \mathbf{0}$. Any movement in θ_t will lead n_t to change at a much faster rate, leading the economy either towards $n_t = 1$ or $n_t = 0$. Hence the time until the economy bifurcates up or down goes to zero as shocks vanish. So the n_t dynamics are basically either $n_\tau^{\uparrow}(t)$ or $n_\tau^{\downarrow}(t)$. The probability it is $n_\tau^{\uparrow}(t)$ approaches $1 - n_\tau$, proportional to the speed at the economy would bifurcate upwards. The converse goes for $n_\tau^{\downarrow}(t)$. So one finds the threshold by finding the indifference point between bifurcating up and down, with their respective probability weights. Figure 1 illustrates θ^*.

Figure 1: Linear Representation of the Decision Thresholds



In the plane, the relative quality of action 1 increases to the right, and the share of individuals choosing action 1 increases upwards. The solid-line threshold is for the case of small shocks, θ^*. Dashed lines show decision thresholds for the most optimistic and pessimistic beliefs about moving to action 1 (e.g., choosing a DSK keyboard).

2 Linear Model: QWERTY *vs.* DSK

To study welfare implications, I use a linear model like in GP. Let actions be to use QWERTY (Q) or DSK (D). Flow utilities are

$$u^Q(\theta_t^Q, n_t) = \theta_t^Q + \nu(1 - n_t)$$

$$u^D(\theta_t^D, n_t) = \theta_t^D + \nu n_t$$

So that the relative utility flow of DSK over QWERTY is $\Delta u(\theta_t, n_t) = \theta_t + \gamma n_t$, with $\theta_t := \theta_t^D - \theta_t^Q - \nu \in \mathbb{R} \equiv \Theta$ and $\gamma := 2\nu > 0$. Notice that, in this specification, the externalities are symmetric for each action. Under small asymmetries in externalities, the qualitative results below still hold.

Proposition 3 *For an economy where the intrinsic relative quality is fixed, the choice thresholds are:*

$$\theta^{opt}(\beta, n_\tau) = -\gamma\left(1 - \Omega(\beta)\right) - \gamma\Omega(\beta)n_\tau \tag{4a}$$

$$\theta^{pes}(\beta, n_\tau) = -\gamma n_\tau \Omega(\beta) \tag{4b}$$

where the notation now makes explicit the dependence of the threshold curves on β *and* $\Omega : [0,1] \to \left[\frac{\rho+\delta}{\rho+2\delta}, \frac{\rho+\delta}{\rho+2\delta}\frac{1-e^{-(\rho+2\delta)}}{1-e^{-(\rho+\delta)}}\right] \subseteq \left[\frac{1}{2}, 1\right]$ *is a strictly decreasing function such that:*

$$\Omega(\beta) = \frac{\rho+\delta}{\rho+2\delta}\frac{1-(1-\beta)e^{-(\rho+2\delta)}}{1-(1-\beta)e^{-(\rho+\delta)}}$$

Proof. One can find these by solving equations (12a) and (12b) for the specific linear utilities. ■

In the $\Theta \times [0,1]$ plane (see Figure 2), the present biased have thresholds flatter than the dynamically consistent. An increase in present bias also brings the two thresholds together, and that is because present biased newborns ask for more quality from the keyboard they expect to become more popular in the future in order to be indifferent between options. One can focus on extreme cases to see exactly why: start with no present bias ($\beta = 1$) and define $\underline{\theta}$ and $\bar{\theta}$ for that economy. Consider a newborn agent who is optimistic about DSK and born at time τ and state $(\theta^{\text{opt}}(1,0), 0)$. If β decreases, the agent sees the externalities coming from QWERTY as more attractive since externalities from DSK will only come in the future. So while that newborn was indifferent under $\beta = 1$, now she strictly prefers QWERTY at that point and will require extra quality to go to DSK. At the other extreme, present bias means little to an optimistic agent born near

$(\theta^{\text{opt}}(1,1),1)$ because she expects little to no change in externalities. Hence the optimistic threshold becomes flatter, but stays the same at $(\underline{\theta},1)$. For the pessimistic threshold, the intuition flips – the newborn expects a migration to QWERTY, so the fixed point in the pessimistic threshold is $(\bar{\theta},0)$. Hence, more present bias induces a decrease in the multiple equilibria area in $\Theta \times [0,1]$.[5]

Now I can combine equations (4a) and (4b) to get the unique threshold in the case with small shocks. I will focus on this threshold for the rest of the paper.

Proposition 4 *When shocks and trends are vanishing, the unique equilibrium-defining threshold is:*

$$\theta^*(\beta, n_\tau) = -\gamma\,(1-\Omega(\beta)) - \gamma\,(2\Omega(\beta)-1)\,n_\tau \tag{5}$$

Proof. Proposition 9, the unique threshold is a linear combination of (4a) and (4b). ■

There are three main reasons to focus the analysis on this "small shocks" threshold. First, it captures the idea that economic fundamentals are always somewhat uncertain and moving around. Second, it allows one to focus on the core issue of coordination problems (the externalities) instead of movements in intrinsic quality. Third, its close relation to the case with no shocks: θ^* is confined between the fixed-θ thresholds and they share the points $(\theta^*(\beta,0),0)$ and $(\theta^*(\beta,1),1)$ (see Figure 2). Increases in present bias bring all of these thresholds even closer together.

When $\beta = 1$ we have no present bias in preferences and the equation simplifies to:

$$\theta^*(1, n_\tau) = -\frac{\gamma\delta}{\rho+2\delta} - n_\tau\frac{\gamma\rho}{\rho+2\delta} \tag{6}$$

so introducing present bias with $0<\beta<1$ induces a rotation of the small shocks threshold around the point $\left(\theta^*\left(1,\frac{1}{2}\right),\frac{1}{2}\right)$. Intuitively, that must happen because, due to the agent's impatience, she will ask for more intrinsic quality from the network she expects society to adopt in the future. The flipping point happens exactly at $n_t = 1/2$ because at that point: (i) the bifurcation probabilities are the same and (ii) the expected future externalities of each keyboard balance out (under symmetric externalities).

[5]This result is not exclusive to present bias. Any increase in discount rates would have a qualitatively similar effect.

3 Welfare Analysis

Consider a central planner who weighs every individual equally, makes choices for the newborns, and exponentially discounts utility.[6] GP shows that this planner makes a newborn pick DSK if:

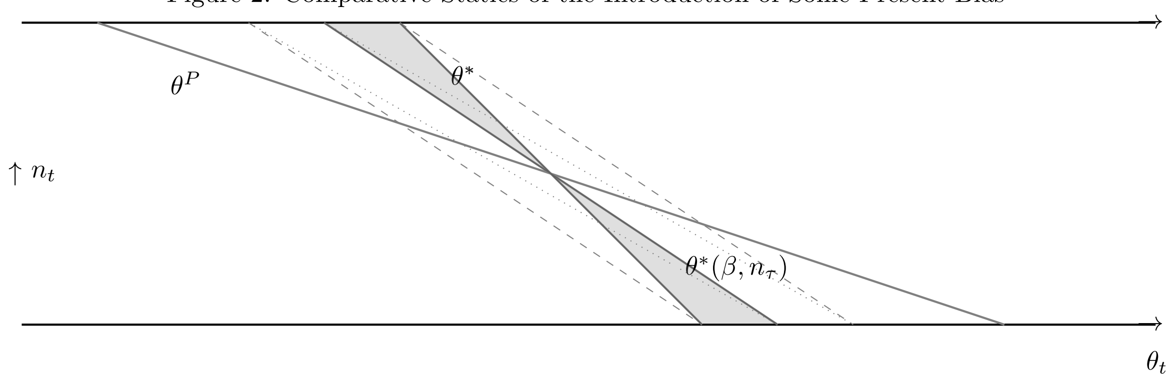
$$\mathbb{E}\int_{\tau}^{+\infty} e^{-(\rho+\delta)(t-\tau)}\left(\underbrace{\theta+\gamma n_t}_{\text{Private }\Delta u}+\underbrace{\gamma\left(n_t-\frac{1}{2}\right)}_{\text{Externality}}\right)dt>0 \tag{7}$$

Intuitively, the decision rule contains the "selfish" flow utility plus the flow externality caused on others. In the case of small shocks, using Proposition 9, the planner's decision threshold is:

$$\theta^P(n_\tau)=\frac{\gamma\rho-2\gamma\delta}{2(\rho+2\delta)}-\frac{2\gamma\rho}{\rho+2\delta}n_\tau \tag{8}$$

The planner's threshold is also a rotation of the individual's threshold around $\left(\theta^*\left(\beta,\frac{1}{2}\right),\frac{1}{2}\right)$. To see that, note that the planner's threshold is half as steep as the unconstrained non-present-biased individuals' threshold. That happens because the planner knows that if a transition to, say, DSK occurs, all individuals locked in QWERTY will be negatively affected by the decreasing QWERTY externality since transitions take time. So, for a transition to be worth it, the planner asks for a higher value of relative DSK quality to compensate.

Figure 2: Comparative Statics of the Introduction of Some Present Bias



Comparison of a society without present bias to a situation with some small amount of present bias. θ^* is the small shocks threshold for a population with no present bias, and $\theta^*(\beta, n_\tau)$ is the one for a population with some present bias, but not enough to create new inefficiency regions. In the shaded area, present bias prevents socially inefficient outcomes. Dashed and dotted lines are the optimistic and pessimistic thresholds under fixed θ for no present bias and some present bias, respectively.

Here, I use the same welfare criterion for the case with present-biased agents (i.e., the planner maximizes the individual utilities, but with $\beta = 1$). This is consistent with a story where the individuals, if they

[6] Implicitly, I assume that the planner cannot change what everybody is doing at once. He can only dictate what an agent chooses when that agent gets a chance to choose.

were able to, would like to be dynamically consistent and discount exponentially, ignoring myopic desires. Alternatively, it is also the decision rule individuals would pick to maximize social utility if they pick the rule i) early enough before the world is put in motion and ii) each individual learns their identity and when they are born.

Now I can state the main result, which says that a little present bias always helps a society follow the optimal path.

Proposition 5 *Under vanishing shocks and trends, for any value of the parameters* $(\rho, \delta, \gamma) \in \mathbb{R}^3_+$*, there is some level of present bias* $\beta \in [0, 1)$ *such that the state space where socially inefficient coordination can occur is strictly smaller (i.e, there is a strictly smaller area in the* $\Theta \times [0, 1]$ *plane in which agents pick the socially inefficient action).*

Proof. $\Omega'(\beta) < 0$ for any $\beta \in [0, 1]$, so any small decrease in β, starting from $\beta = 1$, continuously makes the threshold less steep in the $\Theta \times [0, 1]$ space. As the planner's threshold is always less steep than the dynamically consistent individual's threshold if $(\rho, \delta, \gamma) \in \mathbb{R}^3_+$, then there is always $\tilde{\beta} \in [0, 1)$ such that $2\frac{\partial\theta^*(1,n_\tau)}{\partial n_\tau} \leq \frac{\partial\theta^*(\tilde{\beta},n_\tau)}{\partial n_\tau} < \frac{\partial\theta^*(1,n_\tau)}{\partial n_\tau}$, i.e, $2(1 - 2\Omega(1)) \leq 1 - 2\Omega(\tilde{\beta}) < 1 - 2\Omega(1)$. ■

In fact, if

$$\rho e^{\rho} \leq e^{-\delta}(3\rho + 2\delta + 2e^{-\delta}(\rho + \delta)) \tag{9}$$

then we can find β^* that makes the individuals' threshold exactly equal to the planner's threshold:

$$\beta^* = 1 - \frac{\rho e^{\rho}}{e^{-\delta}(3\rho + 2\delta - 2e^{-\delta}(\rho + \delta))} \tag{10}$$

Equation (9) roughly says that the discount and substitution rates cannot be too high for β^* to exist. In case (9) does not hold, we will see that any level of $\beta \in [0, 1)$ prevents socially inefficient choices. I discuss the reasons for this in Section 4.

If inequality (9) is strict, the present biased individual's threshold can be flatter than the planner's. In this case, there exists a pronounced enough level of present bias such that the coordination in the lower quality action (QWERTY in our example) is inefficient. In this case, different from GP, observing coordination on QWERTY does not imply efficiency.

Figure 2 depicts a situation with some present bias, but not enough to equate the agents' and planner's thresholds (i.e., $\beta < \beta^*$). In the shaded region, present bias keeps society on the efficient path. In other words, if a society found itself in that region and individuals were not present biased, newborns would choose the socially inefficient keyboard, and the society would drift towards coordinating in the "wrong" keyboard. In the area between θ^P and $\theta^*(\beta, n_\tau)$, both present-biased and non-present biased societies choose the "wrong" keyboard.

Some present bias can then internalize some of the social costs of a transition by increasing the importance individuals give to the externalities coming from the currently popular keyboard – DSK must be substantially better than QWERTY to justify a transition under quasi-hyperbolic discount. Hence a society where $\beta < 1$ is kinder to the individuals who would take a long time to adopt DSK in the event of a transition. That is true in the sense that there are fewer situations in which this society will transition to DSK while inefficiently hurting those stuck with QWERTY. Another interpretation of this last result is that if a central planner could not directly make the individual's choices, he would always like individuals to "procrastinate" the network change by asking for more relative intrinsic quality.

Remark 1: infinitely-lived agents interpretation. One can also interpret the results above as an equilibrium with sophisticated infinitely-lived agents that get to reevaluate their choices at rate δ. To see that, model each individual in the society as a succession of selves. As usual in this type of intrapersonal games (e.g, Harris and Laibson (2013)), consider equilibria where agents define strategies only on the payoff-relevant states $(\theta_{t}, _t)$. This precludes past-selves from influencing future selves, because a single player's action does not affect the state of the game. Then the optimal decision rule is still given by equation (11) and the other results follow.

Remark 2: Procrastination. Roughly speaking, procrastinating is consistently postponing something that you *know* you should do. Present biased agents in this model do procrastinate in that sense. To make it clear, consider asking an agent what she would do if *next year* she found herself as a newborn at point $(\theta^*(\beta, 0.1), 0.1)$. That means that the economy would bifurcate upwards with a probability close to 90%, because other people still play according to $\theta^*(\beta, .)$ and shocks are small. In this case, the ex-ante self would *strictly* prefer locking into choosing DSK, because her discount rate for that future decision does not suffer from present bias. Of course, if we consider an agent born at a small fixed distance to the left of that point, then, in the limit when shocks vanish, she would ex-ante lock into QWERTY, because there would be no chance of coordinating on DSK. But while shocks do not completely vanish, that agent thinking about what to choose next year will lock into actions according to a threshold slightly *steeper* than what her society actually follows.

4 When Can Present Bias Be Helpful or Harmful?

Now I analyze when present bias is helpful in that it helps mimic the central planner solution. Specifically, I analyze what range of β can prevent inefficient action as the long-term discount and substitution rates vary. The following proposition creates the formal backbone for the later discussion.

Proposition 6 *Let $\rho > 0$ and $\delta > 0$. Under the small shocks case, suppose $\beta^* \in (0, 1)$, i.e., equation (9) holds with strict inequality. Then*

(i) *For any $\delta > 0$, $\frac{d\beta^*}{d\rho} < 0$, .*

(ii) *For each $\rho > 0$, there exists a unique $\hat{\delta}_\rho$ such that $\frac{\partial\beta^*}{\partial\delta} > 0$ for $\delta < \hat{\delta}_\rho$ and $\frac{\partial\beta^*}{\partial\delta} < 0$ when $\delta > \hat{\delta}_\rho$.*

And we can also show:

(iii) *When $\rho \to 0$, any fixed $\beta < 1$ introduces new inefficiency areas, making a society demand too much quality to start a transition.*

(iv) *For any $\delta > 0$, there is $\hat{\rho}_\delta$ large enough such that for any $\rho > \hat{\rho}_\delta$ any level of present bias prevents inefficient equilibria.*

(v) *For each $\rho > 0$, there is $\underline{\delta}_\rho > 0$ such that present bias is always helpful if $\delta < \underline{\delta}_\rho$*

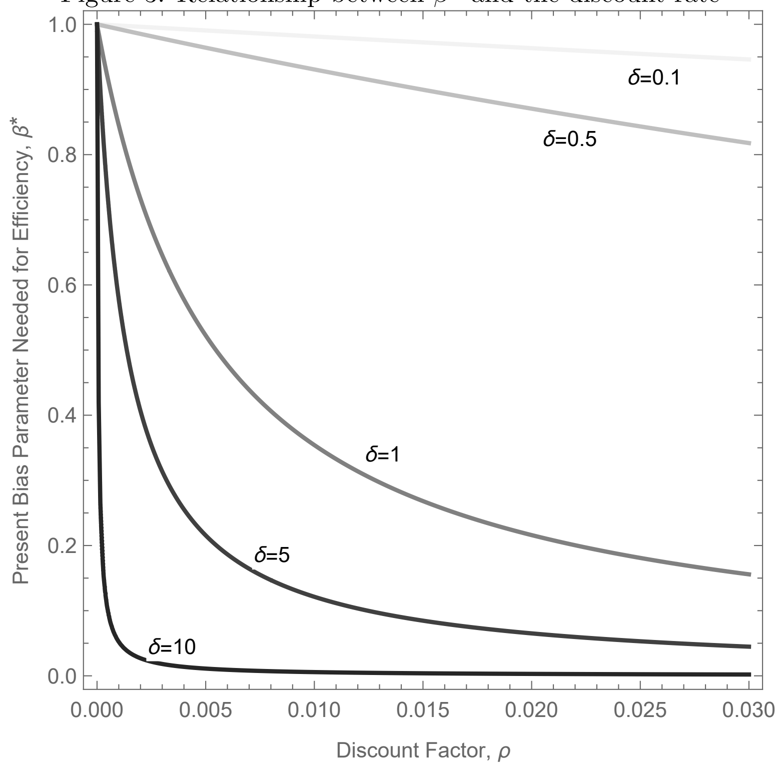
(vi) *For any $\rho > 0$, there is δ large enough such that any level of present bias prevents inefficient equilibria.*

Proof. This relies on examining (9) and (10). See Appendix for details. ■

For what follows, I reparametrize the length of the present to $\frac{1}{365}$, so we can think of the length of the present as one day, and ρ and δ can be interpreted as yearly rates. I define the baseline case with $\rho = 0.02$ and $\delta = 1$, i.e., a long-term discount rate of about 2% a year, with individuals being substituted or reevaluating their choices about once a year. This requires β^* to be nearly 0.22 to equalize the planner's and individual's threshold, and any $\beta \in [0.22, 1)$ would help avoid socially damaging equilibria.

The rate of discount. Figure 3 illustrates how β^* depends on ρ for various levels of the substitution rate. The need for present bias increases with ρ. When $\rho \to 0$, agents are completely patient. In this case, the planner's decision threshold approaches a vertical line (only the long-term matters). Hence, for discount rates close to zero, any introduction of present bias induces too much discount and damages welfare. As ρ increases, the level of present bias that completely internalizes externalities also increases. That happens for two main reasons: (i) an increase in ρ is more important to the planner than for the individual (because he internalizes externalities), so increasing present bias can make the individual's threshold catch up with the planner's, and (ii) a given variation in β becomes less relevant for individual choice when ρ increases (because more discounting happens in the present). So, as ρ increases, more and more present bias is needed to counterbalance increases in ρ and keep the thresholds the same. For ρ high enough, all the curves in Figure 3 reach $\beta^* = 0$, as per Proposition 6. Appendix Figure A1 depicts the same curves for a wider range of ρ.

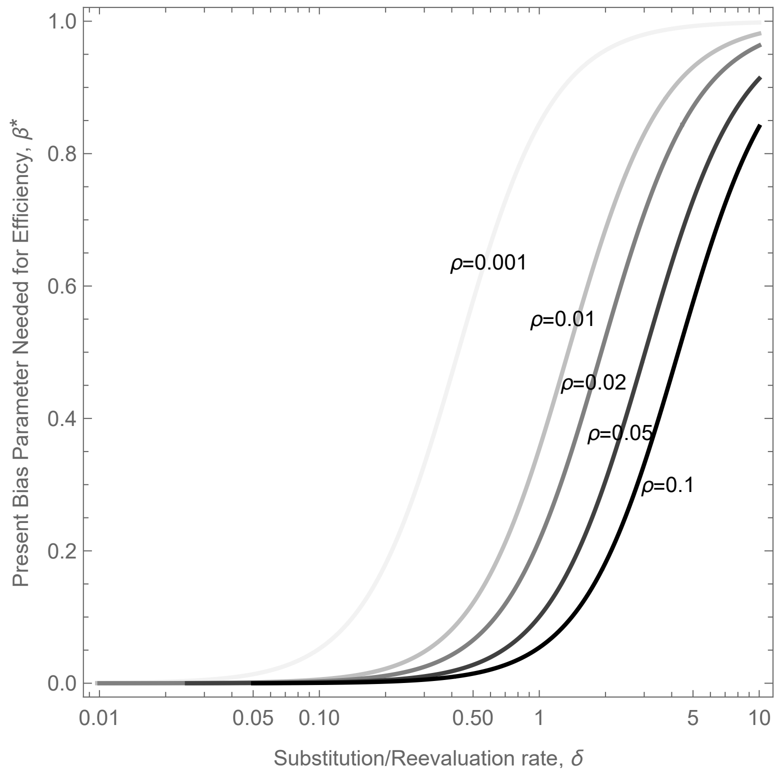
Figure 3: Relationship between β^* and the discount rate



Lines depict – for different values of the substitution/reevaluation rate – the relation between the present bias parameter needed to make the economy efficient, β^*, and the discount rate, ρ.

The substitution/reevaluation rate. In Figure 3, we see that higher substitution rates increase the range in which present bias is helpful when $\delta <= 10$. As per Proposition 5, this happens for "low" values of δ. In Figure 4 we see the relationship between β^* and δ for different values of ρ. When $\delta \to 0$, decisions are "final": agents are stuck with their decisions for a long time and externalities become very important, as transitions take a long time to unravel. Hence, any level of present bias is helpful. As the substitution/reevaluation rate increases, the importance of externalities in transitions decrease and not as much present bias is needed to internalize them, hence the increasing relationship in Figure 4. When $\delta \to +\infty$ frictions vanish, any transition occurs almost instantly. In that case, present bias becomes mostly irrelevant for the agent's choice, exactly because transitions happen instantly, *in the present*. This means present bias has little power to change actions and internalize externalities in such cases, so β^* decreases to 0 for very high substitution rates. This case is less interesting since coordination loses importance. See Figure A2 for how β^* depends on δ over a wider domain.

Figure 4: Relationship between β^* and the friction parameter



Lines depict – for different values of the discount rate – the relation between the present bias parameter needed to make the economy efficient, β^*, and the substitution/reevaluation rate, δ.

Implications. Whether present bias helps or hurts coordination depends on the importance of the future and how fast a society can transition. Present bias is more likely helpful in coordination problems when the long-term discount rate is large and when frictions are significant. If most people rarely reevaluate their keyboard choice and the discount rate is not negligible, present bias helps guarantee the efficient equilibrium. In applications where people frequently reevaluate their choices, present bias is potentially bad, as it prevents socially efficient migrations to better standards. In scenarios where individuals reevaluate choices almost continuously, both present bias and transition externalities trade-offs lose importance.

5 Conclusion

This paper shows that present bias can be incorporated into dynamic coordination problems without significant complications. The main FP results – existence and uniqueness of equilibrium – extend to a setting with present bias. Moreover, in the case utility is linear, externalities are symmetric, and there are vanishing shocks to the relative intrinsic quality, the threshold of a present-biased society is a rotation of the one without present bias.

When shocks to the intrinsic quality of actions are small and frequent, present bias makes people care more about externalities, similar to what a planner would prescribe. In addition, small amounts of present bias can always rule out regions where inefficient choices would occur in the parameter space. However, too much present bias can make individuals overshoot what the planner would like. Finally, present bias is more likely helpful when the discount rate is large and the substitution/reevaluation rate is low.

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Equilibrium Existence

This section introduces the model and characterizes the possible equilibria with a generic utility function. For ease of exposition, I will focus on an overlapping generations formulation. Agents choose their actions when they enter the game. Lifespans have exponential distribution with average δ^{-1}, so δ is the rate at which agents are substituted (henceforth substitution rate). The results will also hold for the case with a fixed measure of (sophisticated or naive) infinitely lived agents who receive chances to rethink their choices at the same rate δ, as argued in Section 3. Most results follow almost immediately from Frankel and Pauzner (2000) (henceforth FP) and Burdzy et al. (1998), so some proofs are either omitted or deferred to the Appendix. For a more comprehensive introduction to the dynamic coordination framework with a similar notation as in this paper, see Guimaraes et al. (2020).

5.1 The Discount Function

I model present bias using a quasi-hyperbolic discount function. From the point of view of an agent at time τ, instantaneous utility at time t is $D(t,\tau)u_t$, where D is the discount function and u_t is the flow utility. In discrete time models, it is common to use discount functions such as $\beta^{\mathbb{1}_{t>\tau}}\Delta^{t-\tau}$, with $(\beta,\Delta)\in[0,1]^2$. When $\beta\in[0,1)$, we have present bias.[7] While there are multiple ways of extending this to continuous time, for simplicity, I use

$$D(t,\tau)=\beta^{\mathbb{1}_{t-\tau\geq 1}}e^{-\rho(t-\tau)}$$

, where $\mathbb{1}$ is the indicator function. In the discount function D, I normalize the length of the present to 1. From the point of view of an agent at time τ, all utility incurred after one unit of time is discounted by β. The length of the present should be thought of as very short (see for instance Ericson and Laibson (2019) and Frederick et al. (2002)). When $\beta=1$, we are left with regular exponential discount at rate ρ.[8]

5.2 The Dynamic Coordination Problem

Time is continuous and homogeneous agents choose between two actions. There is a continuum of individuals $i\in[0,1]$, with lifespans with cumulative density $F(s)=1-e^{-\delta s}$. Newborns choose $a_{it}\in\{0,1\}$ once and

[7] This discount function was introduced by Phelps and Pollak (1968) and widely used later on. See Cohen et al. (2020) for a review of other models of present bias.

[8] See Webb (2016) for more on modelling present bias in continuous time. The results presented here are similar if we use $D(t,\tau,\lambda)=\begin{cases}e^{-(\hat{\beta}/\lambda+\rho)(t-\tau)} & \text{if } t-\tau\leq\lambda\\ e^{-\hat{\beta}-\rho(t-\tau)} & \text{if } t-\tau>\lambda\end{cases}$, with λ as the length of the present.

are substituted when they die. The flow utility of changing to action 1 is $\Delta u(\theta_t, n_t)$, which is continuously differentiable and increasing in both arguments. That is, the benefit of choosing action 1 over 0 increases with the intrinsic relative quality of action 1, θ_t, and with the number of individuals taking action 1, n_t. A newborn at time at time τ with discount function D is uncertain about the path of (θ_t, n_t). So she (the agent) chooses 1 if

$$\mathbb{E}\left[\int_{\tau}^{+\infty} e^{-\delta(t-\tau)} D(t,\tau) \Delta u(\theta_t, n_t)\, dt\right] > 0. \tag{11}$$

5.2.1 Model With No Shocks

Assume that the intrinsic quality θ is fixed through time. We look for the possible equilibria over the set of relative intrinsic qualities $\Theta \subseteq \mathbb{R}$.

Start by assuming the existence of levels of quality extreme enough to make action 1 or action 0 dominant, regardless of what other players do. So there is $\underline{\theta} \in \Theta$ such that if $\theta < \underline{\theta}$, action 0 is dominant for any n_τ, that is, even if an individual believes all newborns will pick 1 from on – the most optimistic belief about 1 –, she will choose 0. Conversely, assume there is $\bar{\theta}$ such that action 1 is dominant if $\theta > \bar{\theta}$. Then we can find an optimistic and a pessimistic threshold, θ^{opt} and θ^{pes}, characterizing all equilibria.

Proposition 7 *There are functions $\theta^{opt} : [0,1] \to \Theta$ and $\theta^{pes} : [0,1] \to \Theta$ such that $\theta^{opt}(n_\tau) < \theta^{pes}(n_\tau)$ for any $n_\tau \in [0,1]$ and:*

- *If $\theta < \theta^{opt}(n_\tau)$, agents always pick 0*
- *If $\theta > \theta^{pes}(n_\tau)$, agents always pick 1*
- *If $\theta^{opt}(n_\tau) < \theta < \theta^{pes}(n_\tau)$, then there are multiple equilibria depending on beliefs*

Proof. See the Appendix. ■

These functions solve:

$$\int_{\tau}^{+\infty} e^{-\delta(t-\tau)} D(t,\tau) \Delta u(\theta^{opt}(n_\tau), n_\tau^{\uparrow}(t))\, dt = 0 \tag{12a}$$

$$\int_{\tau}^{+\infty} e^{-\delta(t-\tau)} D(t,\tau) \Delta u(\theta^{pes}(n_\tau), n_\tau^{\downarrow}(t))\, dt = 0 \tag{12b}$$

where $n_\tau^{\uparrow}(t) = 1 - (1-n_\tau)e^{-\delta(t-\tau)}$ is the most optimistic path for action 1 and $n_\tau^{\downarrow}(t) = n_\tau e^{-\delta(t-\tau)}$ is the most pessimistic. The dashed lines in Figure 1 are linear representations of these thresholds.

The introduction of present bias via the D discount function has no qualitative impact on equilibria characterization under static relative quality. As in FP, two downward-sloping thresholds separate an area of multiple equilibria (between them) from areas with a single equilibrium possibility.

5.2.2 Model With Shocks

Next, I expand FP's result, namely that frequent shocks make the equilibrium unique, to my setting.

$$d\theta_t = \mu\, dt + \sigma dZ_t \tag{13}$$

where Z_t follows a standard Brownian motion. Then we have:

Proposition 8 *Let the trend from the Brownian motion, μ, be a constant. Assume again that dominance regions exist. There is a unique decreasing function $\theta^*(n_\tau)$ such that if $\theta_\tau < \theta^*(n_\tau)$, in equilibrium everyone chooses 0. Conversely, if $\theta_\tau > \theta^*(n_\tau)$, everyone chooses 1.*

This result stems from a contagion argument triggered by the randomness in the path of the relative quality parameter. Complete proofs are found in FP and Guimaraes et al. (2020), but, to build intuition, it is helpful to sketch the argument here.

Since there are dominance regions, start observing that there must be a threshold analogous to θ^{pes} from Proposition 7 for which, under the most pessimistic beliefs about action 1, it is dominant to play 1 to its right. A newborn at the left of that threshold anticipates that others must also play 1 to the right of that threshold, and, given its Brownian motion, θ_t might cross to the right with positive probability.[9] So, taking randomness into account makes playing 1 dominant slightly to the left of that initial threshold. One can then define a second threshold to the left of the first one using the new most pessimistic beliefs about action 1 (i.e., believing others play 1 only to the right of the first threshold). Suppose further that an agent playing according to the second threshold follows the same reasoning as above. In that case, we can define a third threshold to the left of the second, giving rise to an iterative process. This process must converge because the opposite dominance region bounds the sequence of thresholds. Starting from the left, one can make a similar contagion argument, and the proof is finalized by showing both limits coincide (see FP or Guimaraes et al. (2020)).

[9] Agents know other's (time) preferences. This has empirical support from Fedyk (2021).

5.2.3 Small Shocks

Proposition 8 shows that a unique threshold exists under frequent shocks, and now I focus on the case where one can directly calculate that threshold with small shocks.

Proposition 9 *Let $d\theta_t = \alpha\psi(t,\theta_t)\,dt + \sigma dZ_t$. If $(\alpha,\sigma^2) \to \mathbf{0}$, then the threshold θ^* solves:*

$$(1-n_\tau)\int_\tau^{+\infty} e^{-\delta(t-\tau)} D(t,\tau)\Delta u(\theta^*(n_\tau), n_\tau^{\uparrow}(t))\,dt + n_\tau \int_\tau^{+\infty} e^{-\delta(t-\tau)} D(t,\tau)\Delta u(\theta^*(n_\tau), n_\tau^{\downarrow}(t))\,dt = 0 \tag{14}$$

While the proof follows the same steps as in FP and relies on mathematical results from Burdzy et al. (1998), it is worth understanding its intuition. First, notice that if you start on a point on the unique threshold, $(\theta^*(n_\tau), n_\tau)$, while θ_t moves, it does so slowly when $(\alpha,\sigma^2) \to \mathbf{0}$. Any movement in θ_t will lead n_t to change at a much faster rate, leading the economy either towards $n_t = 1$ or $n_t = 0$. Hence the time until the economy bifurcates up or down goes to zero as shocks vanish. So the n_t dynamics are basically either $n_\tau^{\uparrow}(t)$ or $n_\tau^{\downarrow}(t)$. The probability it is $n_\tau^{\uparrow}(t)$ approaches $1 - n_\tau$, proportional to the speed at the economy would bifurcate upwards. The converse goes for $n_\tau^{\downarrow}(t)$. So one finds the threshold by finding the indifference point between bifurcating up and down, with their respective probability weights. Figure 1 illustrates θ^*.

Proofs of Propositions 1 to 3

The proofs follow the arguments in FP. Guimaraes et al. (2020) also presents closely related proofs, with a notation similar to what I use in this paper.

Proof for Proposition 7 As the decision rule is given by 11, this result follows by following the arguments in Section 2.1 from Guimaraes et al. (2020) while replacing the discount function by D. The main insight here is that the thresholds are given the indifference conditions under maximally increasing and decreasing paths of n_t, because those paths make action 1 respectively more and less attractive for future newborns.

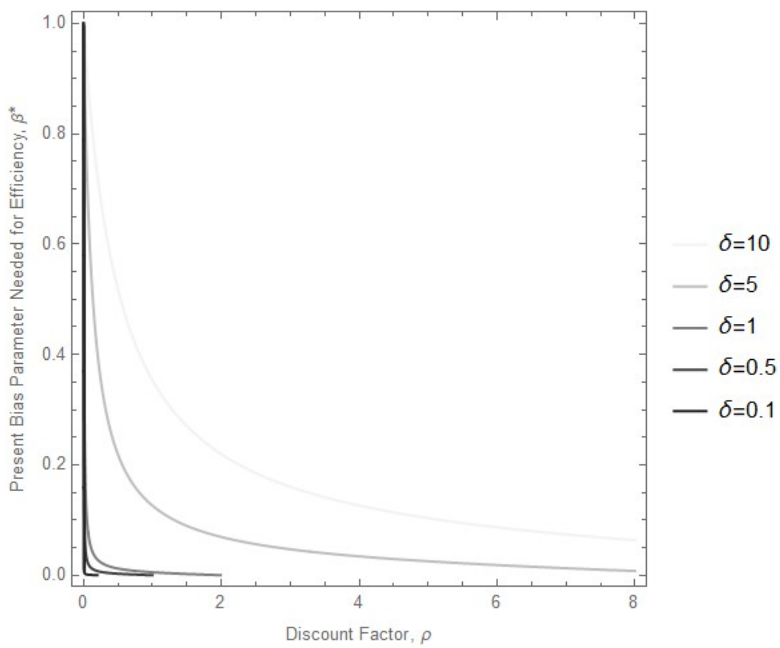
Proof for Proposition 8 The proof for this result is the same as the proof for Theorem 1 in FP or Guimaraes et al. (2020). The main text provides intuition.

Proof for Proposition 9. The proof follows from Theorem 2 from FP along with stochastic bifurcation probabilities results from Burdzy et al. (1998) (i.e., that the time to bifurcation goes to zero as shocks vanish and bifurcation probabilities are proportional to the relative speed a society would bifurcate up or down.)

Proof for Proposition 6. Assume (9) holds with strict inequality. Take the ρ-partial derivative of the LHS of (10). For each $\delta > 0$, the derivative's sign is $sgn(-\rho^2(3-2e^\delta)-(\rho+1)2\delta(1-e^\delta))$, which is negative, so we have (i). Taking the δ-partial derivative of the LHS of (10), we can see it's sign is determined by the sign of $2(e^\delta - 1) + \delta(4-2e^\delta) + \rho(4-3e^\delta)$. Notice that when $\delta \to 0$, the expression determining the sign is positive. The δ derivative of that expression is $4 - 2\delta e^\delta - 3\rho e^\delta$, which is strictly decreasing and eventually strictly negative. This gives us (ii).

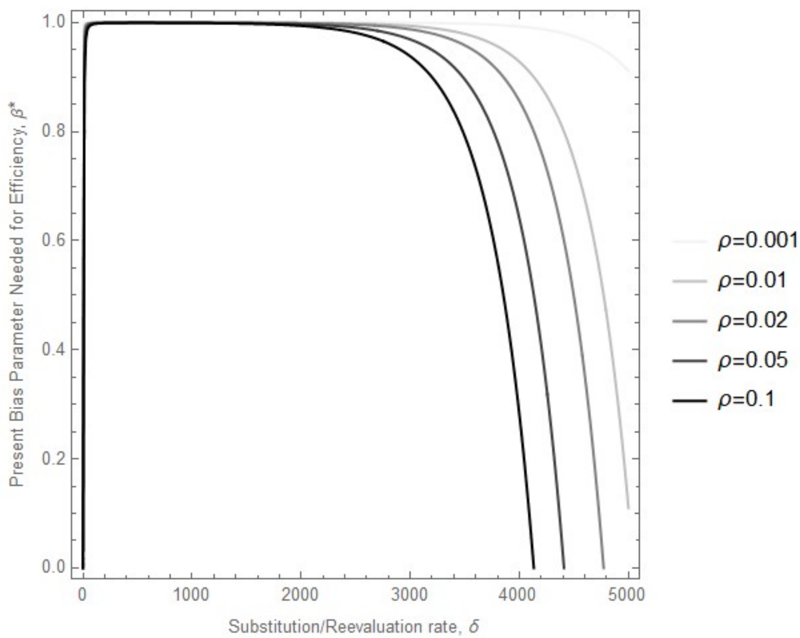
When $\rho \to 0$, $\beta^* = 1$, so (ii) holds. We get (iii) by combining (i) with the fact that letting ρ be big makes (9) false. For (iv), notice (9) does not hold when $\delta \to 0$ since $e^\rho > 1$. To get (vi), just combine the previous steps with the fact (9) does not hold when $\delta \to +\infty$.)

Figure 5: Figure A1: Relationship between β^* and the discount rate



Lines depict – for different values of the substitution/reevaluation rate – the relation between the present bias parameter needed to make the economy efficient, β^*, and the discount rate, ρ.

Figure 6: Figure A2: Relationship between β^* and the substitution/reevaluation rate



Lines depict – for different values of the discount rate – the relation between the present bias parameter needed to make the economy efficient, β^*, and the substitution/reevaluation rate, δ.